作业6

6.1

1 There are 18 mathematics majors and 325 computer science majors at a college.
a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

9 How many different three-letter initials are there that begin with an A?

17 How many strings of five ASCII characters contain the character @ (“at” sign) at least once? [Note: There are 128 different ASCII characters.]

25 How many strings of three decimal digits
a) do not contain the same digit three times?
b) begin with an odd digit?
c) have exactly two digits that are 4 s?

33 How many strings of eight English letters are there
a) that contain no vowels, if letters can be repeated?
b) that contain no vowels, if letters cannot be repeated?
c) that start with a vowel, if letters can be repeated?
d) that start with a vowel, if letters cannot be repeated?
e) that contain at least one vowel, if letters can be repeated?
f) that contain exactly one vowel, if letters can be repeated?
g) that start with X and contain at least one vowel, if letters can be repeated?
h) that start and end with X and contain at least one vowel, if letters can be repeated?

41 A palindrome is a string whose reversal is identical to the string. How many bit strings of length are palindromes?

49 In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if
a) the bride must be next to the groom?
b) the bride is not next to the groom?
c) the bride is positioned somewhere to the left of the groom?

57 Suppose that a password for a computer system must have at least 8 , but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters , and .
a) How many different passwords are available for this computer system?
b) How many of these passwords contain at least one occurrence of at least one of the six special characters?
c) Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes one nanosecond for a hacker to check each possible password.

65 Use the principle of inclusion–exclusion to find the number of positive integers less than 1000000 that are not divisible by either 4 or by 6 .

73 Determine the minimum and the maximum number of matches that can be played in a double–elimination tournament with players, where after each game between two players, the winner goes on and the loser goes on if and only if this is not a second loss.

6.2

1 Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

9 Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

19 A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?

33 Show that there are at least six people in California (population: 39 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials.

41 Find the least number of cables required to connect 100 computers to 20 printers to guarantee that every subset of 20 computers can directly access 20 different printers.

(Here, the assumptions about cables and computers are the same as in Example 9.) Justify your answer.

6.3

1 List all the permutations of .{A,B,C}

9 How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?

19 A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes
a) are there in total?
b) contain exactly two heads?
c) contain at most three tails?
d) contain the same number of heads and tails?

25 How many ways are there for four men and five women to stand in a line so that
a) all men stand together?
b) all women stand together?

33 The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
a) exactly one vowel?
b) exactly two vowels?
c) at least one vowel?
d) at least two vowels?

41 How many license plates consisting of three letters followed by three digits contain no letter or digit twice?

A circular r–permutation of n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.

6.4

1 Find the expansion of $(X+Y)^4$

a) using combinatorial reasoning, as in Example 1.

b) using the binomial theorem.

9 What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$

17. What is the row of Pascal's triangle containing the binomial coefficients $\binom{9}{k}, 0 \leq k \leq 9$?

25. Prove that if n and k are integers with $1 \leq k \leq n$, then $k\binom{n}{k} = n\binom{n-1}{k-1}$,

a) using a combinatorial proof. [Hint: Show that the two sides of the identity count the number of ways to select a subset with k elements from a set with n elements and then an element of this subset.]

b) using an algebraic proof based on the formula for $\binom{n}{r}$ given in Theorem 2 in Section 6.3.

41 Use Exercise 37 to prove the hockeystick identity from Exercise 31. [Hint: First, note that the number of paths from $(0,0)$ to $(n+1,r)$ equals $\binom{n+1+r}{r}$. Second, count the number of paths by summing the number of these paths that start by going k units upward for $k = 0, 1, 2, \ldots, r$.]

6.5

1.In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed?

9. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose

a) six bagels?

b) a dozen bagels?

c) two dozen bagels?

d) a dozen bagels with at least one of each kind?

e) a dozen bagels with at least three egg bagels and no more than two salty bagels?

17. How many strings of 10 ternary digits (0,1 , or 2) are there that contain exactly two 0 s, three 1 s, and five 2 s?

25. How many ways are there to distribute 12 distinguishable objects into six distinguishable boxes so that two objects are placed in each box?

33 How many different strings can be made from the letters in ABRACADABRA, using all the letters?

41 How many ways are there to travel in xyz space from the origin $(0,0,0)$ to the point $(4,3,5)$ by taking steps one unit in the positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative x, y , or z direction is prohibited, so that no backtracking is allowed.)

57. How many ways are there to distribute six indistinguishable objects into four indistinguishable boxes so that each of the boxes contains at least one object?

6.6

1. Place these permutations of $\{1,2,3,4,5\}$ in lexicographic order: 43521, 15432, 45321, 23451, 23514, 14532, 21345, 45213, 31452, 31542 .

9 Use Algorithm 3 to list all the 3–combinations of $\{1,2,3,4,5\}$.

17. Develop an algorithm for producing all permutations of a set of n elements based on the correspondence described in the preamble to Exercise 14.

Chapter 6–Test 1

1. Each locker in an airport is labeled with an uppercase letter followed by three digits. How many different labels for lockers are there?
2. There are 805 lockers in the athletic center and 4026 students who need lockers. Therefore, some students must share lockers. What is the largest number of students who must necessarily share a locker?
3. Find the value of each of the following quantities.
   (a) $C(5,4)$
   (b) $C(5,0)$

(c) $P(5,1)$

(d) $P(5,5)$

4. How many rows are found in a truth table involving nine different propositions?
5. What is the coefficient of x^2y^7 in $(x+y)^9$?
6. How many ways are there to choose five doughnuts if there are eight varieties (and only the type of each doughnut matters)?
7. How many different string can be made using all the letters in the word GOOGOL?